

Effect of Weak Disorder in the Fully Frustrated XY Model.

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The critical behaviour of the Fully Frustrated XY model in presence

of weak positional disorder is studied in a square lattice by Monte Carlo methods. The critical exponent associated to the divergence of the chiral correlation length is found to be $\nu \simeq 1.7$ already at very small values of disorder. Furthermore the helicity modulus jump is found larger than the universal value expected in the XY model.

The study of the fully frustrated XY (FFXY) model^{1,2} in a planar lattices has attracted much attention since it provides a simple model for many physical systems. In particular it is related to Josephson-junction arrays and superconducting wire networks in a magnetic field³ for which recent experiments show interesting properties⁴. In these systems a perpendicular magnetic field B penetrates the array inducing a quenched vortex density. When the field is such that each plaquette of area S in the planar lattice traps half a vortex, e.g. the ratio of the magnetic field to the flux unit $f = BS/\phi_0 = 1/2$, the system can be described by the FFXY model. As first noted by Villain¹ the FFXY model posses, besides the continuous $U(1)$ symmetry, a discrete Z_2 symmetry which is not present in the unfrustrated XY model. The existence of two different symmetries leads to the possibility of two critical temperatures that have been investigated by both analytical methods^{2,5} and Monte Carlo (MC) simulations^{6–10}. However, in spite of accurate MC simulations, the scenario is not fully understood. In fact, there are numerical evidences supporting both the existence of two very close critical temperatures ($T_c^{Z_2} > T_c^{U(1)}$) with critical behavior typical of Ising and Thouless-Kosterlitz-Berezinski (TKB) transitions respectively^{9–11} and the existence of a single transition with novel critical behavior⁷. In the latter case it has been claimed that the transition associated

to the Z_2 symmetry is characterized by a correlation length critical exponent ν lower than that expected in the Ising case.

Recent experiments on Nb wire networks⁴ in an external magnetic field have shown that the critical exponent ν which can be extracted by the scaling of current-voltage (I-V) characteristics, is in agreement neither with a Ising-like exponent ($\nu = 1$) nor with the value suggested in Ref.[7] ($\nu \simeq 0.83$). On the contrary, the scaling of the experimental data suggests a much larger value.

The aim of the present work is to investigate if the effect of weak quenched disorder in the FFFXY model is able to change the critical behaviour of the system and to explain the unexpected critical behaviour.

The hamiltonian of the FFFXY model is

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j - A_{i,j}), \quad (1)$$

where θ_i is the phase in the site i and

$$A_{i,j} = \frac{2\pi}{\phi_0} \int_i^j \vec{A} \cdot d\vec{l} \quad (2)$$

is the line integral of the vector potential along the line $(i - j)$ and ϕ_0 is the flux quantum. In order to simulate positional disorder we consider a uniform distribution of the $A_{i,j}$ whose average value $\langle A_{i,j} \rangle$ is equal to π every second column and zero otherwise. The disorder configuration is thus specified by the probability distribution

$$p(A_{i,j}) = \begin{cases} \frac{1}{2\phi_m} & -\phi_m < (A_{i,j} - \langle A_{i,j} \rangle) < \phi_m \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The parameter ϕ_m controls the degree of randomness and is related to the variance according to $\sigma^2 = \phi_m^2/3$.

Of course, the presence of positional disorder effects the behaviour of the model. In fact it has been shown by renormalization group techniques¹² and Monte Carlo simulations¹⁴ that the critical temperatures associated to both Z_2 and $U(1)$ symmetries decrease with increasing disorder. It has been also suggested for the $U(1)$ transition a reentrant behaviour, even if

recent work on the disordered unfrustrated XY model seems to exclude this possibility¹⁵. However, the critical behaviour that characterize the Ising-like and the TKB transitions have not been studied to our knowledge. In particular, the study of the critical exponent ν associated to the divergence of the coherence lenght of the Ising-like transitions is interesting in view of its possible connection with the $I - V$ characteristics reported by Ling⁴. In fact the experimental data I-V, or equivalently the data J-E, scale very well according to the relation¹⁶

$$E = J |T - T_c|^{\nu z} G(J |T - T_c|^{-\nu}) \quad (4)$$

where the dynamical critical exponent z is found to be about 2 and $\nu = 1.7 \pm 0.5$. This type of scaling is consistent with the reasonable assumption that the relevant excitations that contribute to the critical behaviour of the I-V characteristic are domain-wall.

In order to obtain an estimation of ν we have calculated the Binder's cumulant of the staggered magnetization M

$$U = 1 - \frac{\langle M^4 \rangle}{3\langle M^2 \rangle^2}. \quad (5)$$

where M , associated to the chirality of each plaquette, is defined as

$$M = \frac{1}{L^2} \left| \sum_{\vec{r}} (-1)^{r_x + r_y} m_{\vec{r}} \right|. \quad (6)$$

In eq. (6) the chirality m_r is the rotation of the current around a plaquette

$$m = \frac{1}{\sqrt{8}} (\sin\phi_{12} + \sin\phi_{23} + \sin\phi_{34} + \sin\phi_{41}) \quad (7)$$

where ϕ_{ij} is the phase difference at the edges of the plaquette.

Binder's cumulant allows an accurate estimation of the critical temperature associated to the ordering of the plaquette chirality (Ising-like transition). In fact $U(T_c, L)$ does not depend on lattice size L for large systems and, then, T_c can be identified without making any assumption on the critical exponents. Once a satisfactory estimation of T_c is obtained the critical exponent ν is estimated through a data collapsing with the parameter ν only. The

Binder's cumulant U has been calculated by Monte Carlo (MC) simulations using a standard Metropolis dynamics. For each disorder configuration the mean value $\langle M \rangle$ has been obtained averaging over 4×10^5 MC sweeps after discarding the first 10^5 for equilibrating the system. Then the mean values are further averaged over a number of disorder configurations that range from 200 to 400 depending on lattice size and amount of disorder present in the system.

In Fig. 1) it is shown how the critical temperature $T_c^{Z_2}$ can be extracted by using the MC estimations of U for different lattice sizes. In the case reported in the figure, if we exclude the smallest lattice size, $L = 12$, all the data cross about $T_c^{Z_2} = 0.438 \pm 0.003$. From this type of analysis it is, then, possible to follow $T_c^{Z_2}$ as a function of the disorder (Fig.2)). The results confirm that $T_c^{Z_2}$ decrease with increasing disorder going from $T_c^{Z_2} = 0.452 \pm 0.002$ for the case without disorder ($\phi_m = 0$) to $T_c^{Z_2} = 0.350 \pm 0.005$ for the case with largest disorder studied ($\phi_m = 0.245$). The results are in qualitative agreement with those reported by Choi et al.¹⁴. For larger values of the disorder preliminar results seems to suggest a continous collaps of $T_c^{Z_2}$. In Figs. 3a) and 3b) we show Binder's cumulant U as a function of $|T - T_c^{Z_2}| L^{1/\nu}$ for two values of ϕ_m . By using the estimated values of $T_c^{Z_2}$ at $\phi_m = 0.098$ and $\phi_m = 0.196$ we see that the best data collapsing is obtained for $\nu = 0.9 \pm 0.2$ in the case of lower disorder and for $\nu = 1.7 \pm 0.2$ in the other case. Even if the numerical errors do not allow a very accurate estimation of ν it is very clear that already for very weak disorder ($\phi_m > 0.19$) the critical exponent is quite different from that expected in the Ising model ($\nu = 1$) and from that suggested by Granato et al.⁷ in the case without disorder. Instead the value obtained is very close to that extracted from the $I - V$ characteristics⁴. It is also interesting to note that a very similar value is obtained in the 2d XY spin glass that has a critical temperature $T_c^{Z_2} = 0.17^{-19}$. The same analysis for $\phi_m = 0$ and $\phi_m = 0.245$ gives clearly $\nu \simeq 1$ and $\nu \simeq 1.7$, respectively. On the other hand for the intermediate value $\phi_m = 0.147$ the data are not able to clearly exclude either $\nu \simeq 1$ or $\nu \simeq 1.7$; larger lattices and more MC sweeps are needed to reach a satisfactory estimation of ν . These results indicate the existence of a threshold disorder above which ν changes from 1 to 1.7. The value of the threshold can be, then, estimated to be $\phi_m \simeq 0.19$ but it must be understood

as an upper limit.

As in the FFFXY model we expect the system to present a jump in the helicity modulus at temperature lower or equal to $T_c^{Z_2}$. In order to study this possibility we have calculated the helicity modulus that for the system studied takes the form

$$\begin{aligned} \Gamma = & \frac{J}{L^2} \left\{ -\frac{1}{k_B T} \left\langle \left(\sum_{\langle ij \rangle} \sin(\theta_i - \theta_j - A_{ij}) \hat{e}_{ij} \cdot \hat{x} \right)^2 \right\rangle \right. \\ & + \left\langle \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j - A_{ij}) (\hat{e}_{ij} \cdot \hat{x})^2 \right\rangle \\ & \left. + \frac{1}{k_B T} \left\langle \sum_{\langle ij \rangle} \sin(\theta_i - \theta_j - A_{ij}) \hat{e}_{ij} \cdot \hat{x} \right\rangle^2 \right\} \end{aligned} \quad (8)$$

where \hat{e}_{ij} and \hat{x} are the unitary vectors in the directions of the link $i \mapsto j$ and \vec{x} , respectively. The symbol $\langle \cdot \rangle$ indicates the average over the spin configurations. The elicity modulus Γ has to be averaged further over the disorder configurations to obtain the mean value $\bar{\Gamma}$. Following the analysis developed for the XY model²⁰ and extended to the FFFXY model⁹ we can estimate the critical temperature $T_c^{U(1)}$ by using the following ansatz for the size dependence of $\bar{\Gamma}$

$$\frac{\pi \bar{\Gamma}}{2T_c} = \bar{\Gamma}_0 + \frac{1}{2(\ln L + \ln l_0)} \quad (9)$$

where $\bar{\Gamma}_0 = 1$ corresponds to the universal jump expected at the critical temperature in a KTB transition. This critical scaling, being based on the mapping between a neutral Coulomb gas and a XY model, is valid only if the transition is of KTB type. Therefore eq. (9) can be used as a test for the existence of a standard KTB transition.

In Figs. 4a) and 4b) we show the results of this analysis for two different values of the disorder. In both cases it is not possible to obtain a satisfactory fit of the data by using eq. (9) with $\bar{\Gamma}_0 = 1$. However, a more accurate data analysis shows that a satisfying fitting of the data can be obtained assuming a helicity modulus jump larger than the universal value. From such an analysis we get $T_c^{U(1)} = 0.43 \pm 0.003$ and $\bar{\Gamma}_0 = 1.2$ for $\phi_m = 0.098$ and $T_c^{U(1)} = 0.38 \pm 0.003$ and $\bar{\Gamma}_0 = 1.7$ for $\phi_m = 0.196$. In both cases the fitting parameter l_0 takes the same value found in the FFFXY model without disorder, $l_0 = 0.27$,^{9,13} and $T_c^{U(1)}$ is

slightly smaller than $T_c^{Z_2}$ even if the estimated numerical error does not rule out $T_c^{U(1)} = T_c^{Z_2}$ in the case of lowest disorder ($\phi_m = 0.098$).²¹ This analysis shows that the only way to fit the data with eq.(9) is to assume a larger helicity modulus jump and this is an indication that the transition associated to this jump is not a KTB transition.

Summarizing we have calculated, by MC simulations, the critical temperature and the critical exponent ν associated to the chiral degree of freedom in a disordered FFFXY model. Unexpected we find that the critical exponent ν already for very weak disorder becomes equal to the value found in the XY spin glass, i.e. $\nu \simeq 1.7$. Correspondently, the helicity modulus jump becomes larger than the universal value expected in KTB transition. Finally, we note that the value obtained for ν is in agreement with recent results in Josephson wire networks.⁴

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FIGURE CAPTIONS

Fig. 1 The Binder' s cumulant U (see eq.(5)) vs. temperature T for different sizes L . At the critical temperature U does not depend on L for large L .

Fig. 2 The critical temperature $T_c^{Z_2}$ as a function of the parameter ϕ_m that controls the degree of disorder present in the system. The continous cirve is a guide for the eye only.

Fig. 3 The Binder's cumulant (see eq.(5)) as a function of $(T - T_c^{Z_2})L^{(1/\nu)}$ for two values of the parameter that controls the disorder ϕ_m . In Fig.3a) the data for $\phi_m = 0.098$ are reported having choosen $T_c^{Z_2} = 0.438$ and $\nu = 1$. In Fig.3b) the data for $\phi_m = 0.196$ are reported having choosen $T_c^{Z_2} = 0.395$ and $\nu = 1.7$. Temperature is in units of k_B/J .

Fig. 4 The scaled helicity modulus $\frac{\pi\bar{\Gamma}}{2T_c}$ as a function of $\ln L$ for two values of the parameter that controls the disorder ϕ_m . In Fig.4a) the data for $\phi_m = 0.098$ and $T = 0.45, 0.44, 0.43, 0.42$ (from below to the top) are reported. The best fit is given by the full curve that corresponds to eq.(9) with $\bar{\Gamma}_0 = 1.2$ and $l_0 = 0.27$. In Fig.4b) the data for $\phi_m = 0.196$ and $T = 41, 39, 38, 37$ (from below to the top) are reported. The best fit is given by the full curve that corresponds to eq.(9) with $\bar{\Gamma}_0 = 1.7$ and $l_0 = 0.27$.







